1.5 Scientific Notation Operations

Now that we understand what scientific notation is, we can begin performing operations with these numbers just as scientists have to in their research.

Multiplication and Division with Scientific Notation

Let's say the debt in the United States is now near 1.8×10^{13} dollars and that there are about $300{,}000{,}000$ people in the United States. If every person paid their fair share of the debt, approximately how much would that be per person? To solve this problem we need to divide the debt between all the people. Let's start by writing the standard form number in scientific notation.

$$300{,}000{,}000 = 3 \times 10^8$$

Now we can begin to divide those scientific notation numbers by writing the problem in fraction form:

$$\frac{1.8 \times 10^{13}}{3 \times 10^8}$$

How do we actually divide these? Since the only operations happening are multiplication and division, we can divide the whole numbers and then divide powers of ten. Why does this work? Because we can actually split this into two different fractions and simply each fraction as follows:

$$\frac{1.8 \times 10^{13}}{3 \times 10^8} = \frac{1.8}{3} \times \frac{10^{13}}{10^8} = 0.6 \times 10^5$$

The problem now is that our answer is not in scientific notation. We need to be a single non-zero digit before the decimal. We could multiply the six tenths by ten to make it six, but remember with any expression we can't just change values. The only thing we can multiply or divide by is one. So if multiply by ten, we have to divide by ten so that there is no change in the actual value of the number, like so:

$$0.6 \times 10^5 = 0.6 \times (10 \div 10) \times 10^5 = (0.6 \times 10) \times (10^5 \div 10) = 6 \times 10^4$$

While this appears confusing, it really just keeping everything balanced. Since we made the 0.6 bigger by a factor of 10, we have to make the 10^5 smaller by a factor of 10 to balance it.

Let's look at another example. There are approximately 6×10^9 people in the world. If each person made 2.5×10^4 dollars in a year, how much money was made worldwide? In this case we need to multiply the numbers, rearranging using the commutative property as follows:

$$(6 \times 10^9)(2.5 \times 10^4) = 6 \times 2.5 \times 10^9 \times 10^4 = 15 \times 10^{13}$$

Again, we need to get our answer in scientific notation. In this case, the 15 needs to be a 1.5, meaning it needs to be smaller a single factor of 10. Therefore we will have to make the 10^{13} bigger by a factor of ten making it 10^{14}. This means our final answer is 1.5×10^{14} dollars produced in that year.

Addition and Subtraction with Scientific Notation

When adding or subtracting numbers in scientific notation, we have to remember that place value matters. For example when adding 123 and 56, we have to add the 6 and 3 because they are both in the ones place. In the same way, we have to add the 2 and 5 because they are both in the tens place.

Since scientific notation makes every number a single digit followed by a decimal and a power of ten, the place value gets hidden. This means that we can only add or subtract scientific notation numbers if they have the same power of ten (since the power of ten controls the place value). If they don't have the same power of ten, we will have to rewrite one of the numbers in a way such that the powers of ten are equal.

For example, let's solve the problem $3.7 \times 10^6 + 4.3 \times 10^5$ where the powers of ten are different. First we need to make the powers of ten be the same. Since 3.7×10^6 is the larger number, we'll leave that alone and convert 4.3×10^5 into an equal number that has $\times 10^6$ at the end. Since we want the power of ten to be bigger by a single factor of ten, we'll need to make the 4.3 smaller by a factor of ten as follows:

$$4.3 \times 10^5 = 0.43 \times 10^6$$

Now that we have the same power of ten (which means the same place value), we can solve as follows:

$$3.7 \times 10^6 + 0.43 \times 10^6 = 4.13 \times 10^6$$

Notice that we simply added the 3.7 and the 0.43 together. Notice that we could have turned the scientific notation numbers into standard form numbers and then added. However, this is only convenient with small powers of ten.

$$3.7 \times 10^6 = 3{,}700{,}000$$

$$4.3 \times 10^5 = 430{,}000$$

$$3.7 \times 10^6 + 4.3 \times 10^5 = 3{,}700{,}000 + 430{,}000 = 4{,}130{,}000 = 4.13 \times 10^6$$

Estimating Very Large Numbers with Powers of 10

Scientists have measured the temperature at the edge of the sun to be around $5{,}400°\ C$. Let's round that number to a single digit times a power of ten. Looking at the leftmost digit, which is 5, should that stay a 5 or round up to 6? Since the next number is only a four, it will stay a 5. That means $5{,}400 \approx 5{,}000$. Now let's look at our place value.

This shows us that we can write $5{,}000$ as 5×10^3. We may be tempted to simply count the number of zeros and use that as the power, but that will only work for this specific type of problem. Instead we should continue to think about place value.

Let's look at one final example of rounding a large number to a single digit times a power of ten. In a penny there may be approximately $19{,}370{,}000{,}000{,}000{,}000{,}000{,}000$ atoms.

$$19{,}370{,}000{,}000{,}000{,}000{,}000{,}000 \approx 20{,}000{,}000{,}000{,}000{,}000{,}000{,}000$$

$$20{,}000{,}000{,}000{,}000{,}000{,}000{,}000 = 2 \times 10^{22}$$

$$19{,}370{,}000{,}000{,}000{,}000{,}000{,}000 \approx 2 \times 10^{22}$$

So there are approximately 2×10^{22} atoms in a penny. Notice that we could simply count the place values from right to left starting from the where the decimal would be to find our exponent. In other words, we could count how many places the decimal "moved" to get the new number.

Estimating Very Small Numbers with Powers of 10

This will work the same way except that now we will have negative powers of ten since we will be dealing with very small decimals. For example, a single atom in that penny is approximately 0.0000000312 cm across. We can round that using powers of ten in nearly the same way.

First, look at the leftmost non-zero digit, which is a 3. Should that stay a 3 or should it round up to a 4? It should stay a 3 because the next digit is a one and we only round up when the number is five or greater. That means that $0.0000000312 \approx 0.00000003$. Now let's examine that in our place value chart. Notice that we have negative exponents because the tenths place is really the fraction $\frac{1}{10}$ which is 10^{-1} and so forth.

Now we see that $0.0000000312 \approx 3 \times 10^{-8}$.

Let's do one final example of rounding a small number using a single digit times a power of ten. Round the number $0.000\ 000\ 000\ 000\ 000\ 871$ to single digit times a power of ten. (Notice that sometimes we put spaces between every three zeros to make it easier to count how many zeros are there.)

$$0.000\ 000\ 000\ 000\ 000\ 871 \ \approx 0.000\ 000\ 000\ 000\ 000\ 9$$

$$0.000\ 000\ 000\ 000\ 000\ 9 = 9 \times 10^{-16}$$

$$0.000\ 000\ 000\ 000\ 000\ 871 \ \approx 9 \times 10^{-16}$$

Notice again that we could simply count the number of places the decimal "moved" to make it 9. That took 16 places for the decimal to move; therefore we will use the negative 16th power as our exponent.

Estimating Operations

When faced with an operation involving scientific notation, we can estimate the final solution by rounding the numbers first then performing the operation. This will make your final solution an estimate as well.

Lesson 1.5

Compute the EXACT answer to each of the following questions giving your answer in scientific notation.

1. $(3 \times 10^{-6})(3 \times 10^{9})$

2. $\frac{6.8\times10^{9}}{2\times10^{5}}$

3. $4.5 \times 10^{7} + 41{,}000{,}000$

4. $8.4 \times 10^{7} - 3.1 \times 10^{7}$

5. $(2.4 \times 10^{4})(3{,}000)$

6. $\frac{5.4\times10^{8}}{3{,}000}$

7. $3.9 \times 10^{13} + 4.2 \times 10^{13}$

8. $8.2 \times 10^{-5} - 0.000\,059$

9. $(1.3 \times 10^{-4})(4.2 \times 10^{11})$

10. $\frac{4.5\times10^{9}}{1.5\times10^{13}}$

11. $1.3 \times 10^{7} + 4 \times 10^{7}$

12. $5.2 \times 10^{7} - 12{,}000{,}000$

ESTIMATE the answer to each of the following questions giving your answer as a single digit times a power of ten.

13. The distance from the Earth to the sun is $9.3 \text{ X } 10^6$ miles. The distance from the Earth to the moon is $3 \text{ X } 10^5$ miles. How many times bigger is the distance from Earth to the sun than the distance from Earth to the moon?

14. The temperature halfway to the Sun from Mercury is approximately $1{,}800° \, C$ and scientists theorize that it may be up to $26{,}000$ times hotter at the center of the Sun. Approximately how hot is it at the center of the Sun?

15. Each shrimp weighs approximately $0.000\ 27\ g$ and a shrimp company can bring in over $3{,}100{,}000{,}000$ shrimp per year. Approximately how much would that many shrimp weigh?

16. The Earth has a mass of about 1×10^{25} kg. Neptune has a mass of 1.8×10^{27} kg. How many times bigger is Neptune than Earth?

17. A country has an area of approximately $8{,}400{,}000{,}000$ square miles and has approximately $210{,}000$ people. How much area is this per person?

18. A blue whale can eat $300{,}000{,}000$ krill in a day. All of that krill weighs approximately $6{,}300{,}000{,}000\ mg$. About how much does each krill weigh?

19. The US spends on average $10{,}200$ dollars on each student per year. There are about $77{,}000{,}000$ students in the United States. How about much money total is spent on students each year?

20. McDonald's has about $210{,}000$ managers and each makes on average $39{,}000$ dollars per year. How much money does McDonald's spend on managers each year?